PHYSICAL REVIEW A **72**, 060304(R) (2005)

Generalized universal cloning and purification in quantum information by multistep state symmetrization

L. Masullo, M. Ricci, and F. De Martini*

Dipartimento di Fisica and Consorzio Nazionale Interuniversitario per le Scienze Fisiche della Materia, Università di Roma "La Sapienza," Piazza Aldo Moro 5, Roma I-00185, Italy

(Received 21 April 2005; revised manuscript received 2 August 2005; published 9 December 2005; publisher error corrected 13 December 2005)

A general multistep linear state symmetrization device for photonic qubits is presented together with the experimental realizations of the $1\rightarrow 3$ and $2\rightarrow 3$ universal optimal quantum cloning machines and of a 3-qubit purification procedure. Since the present method exploits the bosonic nature of the photons, it can be applied to any particle obeying to the Bose statistics. On a technological perspective, the present protocol is expected to find relevant applications as a multiqubit symmetrization device to be used in modern quantum-information networks.

DOI: 10.1103/PhysRevA.72.060304 PACS number(s): 03.67.−a, 42.50.Dv

In recent years quantum optics turned out to be an excellent experimental test bench for several new quantum-information (QI) concepts. In this scenario, linear optics (LO) has revealed all its power by allowing the realization of several relevant communication and computational protocols. In particular, during the last years, LO has been exploited to implement the state symmetrization of photonic qubits, where symmetrization means the projection of qubits on the symmetric subspace (SSP) [1–3]. This process lies at the heart of several relevant QI protocols. For instance, the probabilistic $N\rightarrow M$ universal and optimal quantum cloning machine (UOQCM) exploits the action of SSP acting on N identical input qubits and $(M-N)$ fully mixed ancillas in order to uniformly and optimally distribute the initial information into the overall symmetric output state consisting of M identical qubits [1,3]. In addition and quite remarkably, it has been proposed that SSP can be adopted to suppress or prevent noise affecting quantum computation and communication [2,4–6], e.g., to *purify* single qubits during the transmission over a noisy quantum channel.

Several of these protocols have been recently implemented involving two polarization (π) encoded photonic qubits where the SSP was carried out by means of LO techniques [7–11]. Precisely, if two independent π qubits impinge onto the input arms of a beam splitter (BS) in an Hong-Ou-Mandel (HOM) interferometer [12], SSP is probabilistically realized by postselecting the events in which the two photons emerge in the same spatial output mode. The basic principle at the heart of these realizations is the following: the two photons are initially superimposed at the BS interface in order to make them indistinguishable; then, by postselection, a spatial symmetric wave function of the two photons is imposed. This assures a symmetric wave function for the polarization degree of freedom too, i.e., the qubits space, in compliance with the bosonic nature of photons. In spite of the relevance of the 2-qubit SSP operation, so far no viable experimental solution was found to extend these protocols to involve in a controlled way more photons in the game, i.e., $N>1$ and $M>2$ in the cloning process and $M>2$ in the purification process. On the other hand, this issue is positively a highly significant one. Besides their conceptual relevance *per se*, multiple cloning and purification are at the basis of the recent proposal of superbroadcasting of mixed states, which can be found in a wide application in distributing QI in a noisy environment [13]. Moreover, the $1\rightarrow M$ cloning process naturally establishes quantum channels that can be used for multipartite communication tasks. For instance, this transfer of a localized information into multiqubit correlations provides secure strategies to *hide* reversibily QI. Indeed that can only be retrieved when a specific classical combination is available [14].

In the present work we generalize the above 2-qubit LO scheme by a chain of interconnected HOM interferometers, each one consisting of a 50/50 BS and realizing the qubit symmetrization in Hilbert spaces of increasing dimensions [15]. The device has been applied to assemble a $1\rightarrow 3$ UOQCM as a cascade of a $1\rightarrow 2$ and a $2\rightarrow 3$ machines [Fig. 1(a)]; by this procedure, we have experimentally demonstrated the concatenation property of the universal cloning [16], a fundamental step for a better comprehension of the QI flux underlying this process. The validity of the protocol has been further tested by the realization by a two-step chain of the $M=3$ qubits purification task. In full analogy with the 2-qubit case, the HOM-chain apparatus exploits the bosonic statistics of photons to accomplish the SSP operation without employing auxiliary resources: whenever the M input photons of the chain become indistinguishable, SSP is probabilistically realized by postselecting when *all* photons emerge in the *same* spatial output mode. The HOM chain is designed to make indistinguishable all the M independent π qubits involved; this is done precisely by a chain of $(M-1)$ intermediate processes, each one accomplishing this task for a single qubit starting from the two input photons at BS_1 interference. Unless the last BS step, at any intermediate links of the chain, half of the signal is lost since it can emerge from the unconnected output modes [e.g., k_b in Fig. 1(b)]. Actually, the resulting nonunit success probability of the "indistinguishability" procedure, $p=2^{-(M-2)}$, represents the cost to *erase* the spatial information about different qubits. To

*Corresponding author: francesco.demartini@uniroma1.it

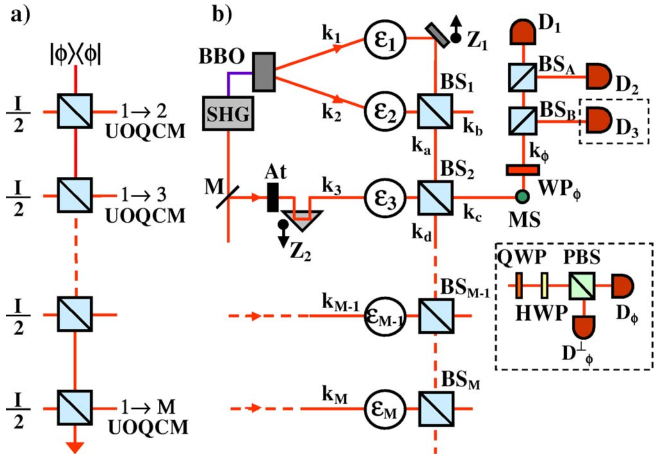

FIG. 1. (Color online) (a) Linear optical scheme of a multistep state symmetrization device realized by a chain of identical beam splitters (BS) and adopted to implement the $1\rightarrow M$ UOQCM. (b)Experimental setup involving a two-step chain. In the figure: ε_i refers to the qubit preparation stage; MS is the mode selector realized by a single mode optical fiber. The figure includes a possible M-step generalization of the process. In the inset it is shown the apparatus for quantum state tomography.

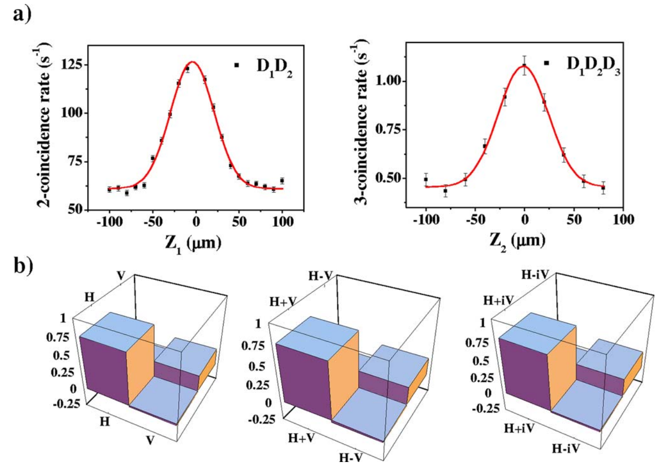

FIG. 2. (Color online) (a) The two- and three-coincidence peaks identify the full resonance $Z_1=0$, $Z_2=0$ conditions of the system, i.e., the realization of simultaneous HOM interference involving both BS_1 and BS_2 devices. The visibility, V_1, of the BS_1-if involving SPDC pairs in the product state $|H\rangle_1|H\rangle_2$ was typically 0.97 respect to an ideal value of 1. The visibility, V_2, of the BS_2-if corresponding to a three-photon interference (the two-photon state $|HH\rangle_a$ from the output of BS_1-if and the faint laser pulse $|H\rangle_3$) was typically 1.20 to be compared with the expected value 2. This deviation from the theoretical value is due to the nonperfect indistinguishability between photons from different sources. (b) Quantum state tomography (QST) of the density matrix of the output cloned states drawn in the diagonal basis of the input states. Only the real part contributions of the QST diagrams are reported while the corresponding imaginary parts have been found to be negligible.

illustrate the application of the device, let us consider as a paradigmatic example the $1\rightarrow 3$ cloning process, Fig. 1(a). The procedure prescribes at the first step the symmetrization of the input π qubit $|\phi\rangle$ to be cloned with a fully mixed ancilla at BS_1. As said, this step is accomplished whenever the two qubits emerge in the same output mode k_a or, alternatively, k_b. Then, the two-photon output of this machine belonging to the mode k_a, i.e., the output of a $1\rightarrow 2$ UOQCM, provides the input of the $2\rightarrow 3$ device realized at BS_2 together with another mixed ancilla on k_3. After a successful application of the SSP operation we attain as final output three optimal clones of $|\phi\rangle$.

Experimental setup. The main laser source of the apparatus was a Ti:sapphire mode-locked pulsed laser with wavelength $\lambda=795$ nm and repetition rate 76 MHz. The input qubits ρ_1 and ρ_2 were provided by a pair of photons generated by spontaneous parametric down conversion (SPDC) in a 1-mm-thick BBO crystal, cut for type-I phase matching and pumped by an UV laser beam ($\lambda_{UV}=397.5$ nm, $P=300$ mW) supplied by a second harmonic generator (SHG) in Fig. 1(b). The single pair creation probability was $\alpha\simeq 0.015$. The two photons, each with wavelength $\lambda=795$ nm and coherence time $\tau_{coh}\simeq 200$ fs, were emitted in a product state of horizontal (H) linear polarizations over the modes k_1 and k_2. These ones were then drawn into a linear superposition in BS_1 interference. The exact space-time overlap of the two input photons implying the actual realization of the HOM interference was controlled by the microscopic BS_1 interference displacement: $Z_1=2c\Delta t_1$. Let us call "BS_1 *interference*" (BS_1-if) the condition $Z_1=0$ corresponding to the maximum overlap. The third qubit, again π codified, was emitted in the quasi single-photon condition over the mode k_3 by high attenuation of a laser beam deflected by the mirror M from the main H-polarized laser. The mean number of photons per pulse was $\bar{n}\simeq 0.05$. The beam was then delayed by $Z_2=2c\Delta t_2$ via an optical "trombone." Analogously to the former case, BS_2-if stands for the condition $Z_1, Z_2=0$ which implies the exact space-time overlap of all qubits at BS_2, i.e., the overall three-photon interference: Fig. 1(b). The qubits were independently π encoded by means of preparation stages $\varepsilon_1,\varepsilon_2,\varepsilon_3$ each one realized by a rotating $\lambda/2$-wave plate, a $\lambda/4$-*wp* and a LiO_3 Pockels cell optionally driven by a high-voltage generator. By these devices any π state, *pure* or *mixed*, could be independently codified on each qubit [7]. The 3-qubit SSP operation was switched on by setting $Z_1=Z_2=0$ and postselecting the events in which three photons bunch up together over the output mode k_c. The identical effect expected on mode k_d was not exploited for simplicity. The necessary high mode selection of the output beam over k_c was provided by a 5 m long single mode optical fiber MS compensated by the wave plate WP_ϕ. By the apparatus consisting of the detectors D_1, D_2, and D_3, coupled to the mode k_ϕ by BS_A and BS_B, the two- and three-photon interference patterns realized at BS_1 and BS_2 were measured versus the displacements Z_1 and Z_2: the HOM-chain working was completely characterized by the visibilities of these interference fringes reported in Fig. 2(a). All adopted D's were equal SPCM-AQR14 Si-avalanche single photon units with quantum efficiency ~ 0.55. One interference filter with bandwidth $\Delta\lambda=4.5$ nm was placed in front of each D. Since after the SSP projection all qubits are described by the same reduced density matrix and belong to the same spatial mode, in order to verify precisely the cloning and/or the purification processes a quantum state tomography (QST) of one of the output π qubits was undertaken. This measurement was realized by replacing the detector D_3 by the QST setup depicted in the inset of Fig. 1(b). This apparatus consisted of a $\lambda/4$ wave plate (QWP), followed by a $\lambda/2$ wave plate (HWP) and by a polarizing BS (PBS). The reduced density matrix of the single qubit was reconstructed by means of the two triple-coincidence sets $\{[D_\phi, D_1, D_2], [D_\phi^\perp, D_1, D_2]\}$ that postselected the SSP operation. By these data the normalized Stoke's parameters of the single output qubit were obtained in terms of the coincidence rates as $([D_\phi, D_1, D_2] - [D_\phi^\perp, D_1, D_2]) \times ([D_\phi, D_1, D_2] + [D_\phi^\perp, D_1, D_2])^{-1}$ for three different settings of the wave plates. A trivial extension of the SSP device to a larger number M of qubits is illustrated in Fig. 1(b): qubits generated by additional, independent single-photon sources, encoded by ε_i are injected over the modes k_i on equal and equally interconnected BS_is for i in the range $1-M$. Let us now return to the specific processes to be investigated.

$1\rightarrow 3$ *universal optimal quantum cloning machine*. In agreement with the previous discussions, the $1\rightarrow 3$ process was implemented by a concatenation of a $1\rightarrow 2$ and a $2\rightarrow 3$ UOQCMs. In a preliminary experiment these two machines have been tested separately. The first device was realized on the BS_1 by sending as input the 2-qubit state: $\rho_{\text{in}}^{1\rightarrow 2} = |\phi\rangle\langle\phi|_1 \otimes I_2/2$ and by postselecting appropriate two-fold coincidence counts between detectors D' s. Three different input states $\rho_1 = |\phi\rangle\langle\phi|$ with $|\phi\rangle = |H\rangle$, $2^{-1/2}(|H\rangle + |V\rangle)$, $2^{-1/2}(|H\rangle + i|V\rangle)$, henceforth referred to as $|H\rangle$, $|H+V\rangle$, and $|H+iV\rangle$ were adopted to test in sequence the universality of the device. The experimental values of fidelity were $F_{1\rightarrow 2}^{H} = 0.831 \pm 0.003$; $F_{1\rightarrow 2}^{H+V} = 0.833 \pm 0.002$; $F_{1\rightarrow 2}^{H+iV} = 0.830 \pm 0.002$ to be compared with the theoretical value: $F_{1\rightarrow 2}^{theor} = 5/6 \simeq 0.833$. Then, the $2\rightarrow 3$ machine was carried out by setting $\rho_1 = \rho_2 = |\phi\rangle\langle\phi|$ and $\rho_3 = I_3/2$. Indeed, for $Z_1 = 0$, over the mode k_a the state $|\phi\rangle\langle\phi|^{\otimes 2}$, the very input of the $2\rightarrow 3$ UOQCM, is realized. By triple coincidence measurements the experimental fidelities $F_{2\rightarrow 3}^{H} = 0.895 \pm 0.003$, $F_{2\rightarrow 3}^{H+V} = 0.893 \pm 0.003$, $F_{2\rightarrow 3}^{H+iV} = 0.894 \pm 0.003$ were attained to be compared with the theoretical value $F_{2\rightarrow 3}^{theor} = 11/12 \simeq 0.917$. Finally, the $1\rightarrow 3$ UOQCM was realized adopting as overall input of the 2-BS chain the state $\rho_{\text{in}}^{1\rightarrow 3} = |\phi\rangle\langle\phi|_1 \otimes \mathbb{I}_2/2 \otimes \mathbb{I}_3/2$. After successful application of the 3-qubit SSP operator, $\Pi_+^{(3)}$, the following symmetrized output is obtained:

$$\rho_{out}^{1\rightarrow 3} = \frac{\Pi_+^{(3)} \rho_{in}^{1\rightarrow 3} \Pi_+^{(3)}}{Tr[\Pi_+^{(3)} \rho_{in}^{1\rightarrow 3} \Pi_+^{(3)}]} = \tfrac{3}{6}|\phi\phi\phi\rangle\langle\phi\phi\phi| + \tfrac{2}{6}|\{\phi\phi\phi^\perp\}\rangle\langle\{\phi\phi\phi^\perp\}| + \tfrac{1}{6}|\{\phi\phi^\perp\phi^\perp\}\rangle\langle\{\phi\phi^\perp\phi^\perp\}| \quad (1)$$

where $|\{m\phi, n\phi^\perp\}\rangle$ stands for a symmetric wave function of m qubits in the state $|\phi\rangle$ and n in the state $|\phi^\perp\rangle$. Each one of the identical clones $j = 1, 2, 3$ can be expressed by the reduced density matrix $\sigma_j^{1\rightarrow 3} = Tr_{h,k\neq j}\rho_{\text{out}}^{1\rightarrow 3} = \frac{7}{9}|\phi\rangle\langle\phi| + \frac{2}{9}|\phi^\perp\rangle \times \langle\phi^\perp|$ and is characterized by the fidelity value: $F_{1\rightarrow 3} = \langle\phi|\sigma_j^{1\rightarrow 3}|\phi\rangle = \frac{7}{9}$. In Eq. (1) $\text{Tr}[\Pi_+^{(3)} \rho_{\text{in}}^{1\rightarrow 3} \Pi_+^{(3)}]$ represents the theoretical success probability of the cloning: $p_{1\rightarrow 3} = \frac{1}{2}$. As previously discussed, the actual theoretical experimental probability was equal to $p_{1\rightarrow 3}^{\text{expt}} = \frac{1}{4}$ because the discarded signal over k_b. As said, we have analyzed only the output mode k_c. The experimental test procedure was as follows: the preparation of ρ_1, ρ_2, and ρ_3 was tested by QST reconstruction and assured by a value of the corresponding Uhlman fidelity $F(\rho_i^{\text{expt}}, \rho_i^{theor}) = (\text{Tr}[\sqrt{\sqrt{\rho_i^{\text{theor}}}\rho_i^{\text{expt}}\sqrt{\rho_i^{\text{theor}}}}])^2$ larger than 0.98; then the states $|H\rangle$, $|H+V\rangle$, $|H+iV\rangle$ were adopted to test the universality of the device as in the previous experiments. The experimental value of the cloning fidelity was directly measured again by QST reconstruction of one of the output clones. The following results: $F_{1\rightarrow 3}^{H} = 0.762 \pm 0.008$, $F_{1\rightarrow 3}^{H+V} = 0.761 \pm 0.008$, $F_{1\rightarrow 3}^{H+iV} = 0.768 \pm 0.008$ have been attained to be compared with the optimum value: $F_{1\rightarrow 3}^{\text{theor}} = 7/9 \simeq 0.777$. The experimentally reconstructed density matrices for the clones were reported in Fig. 2(b). To verify more in depth the concatenation property we have calculated for each UOQCM the corresponding shrinking factors, $\eta(N,M)$, where $\eta(N,M) = 2F_{N\rightarrow M} - 1$ is the length of the clones vector on the Bloch sphere. It was shown that for a cascade of cloning processes η multiplies: $\eta(N,M) = \eta(N,\text{L}) \times \eta(\text{L},M)$ [16]. From the measured fidelities we have derived the averaged values: $\eta^{\text{expt}}(1,2) = 0.663 \pm 0.003$, $\eta^{\text{expt}}(2,3) = 0.788 \pm 0.003$. Thus the $\eta^{\text{expt}}(1,3)$ was expected to be equal to $\eta^{\text{expt}}(1,2) \times \eta^{\text{expt}}(2,3) = 0.522 \pm 0.003$. From the $1\rightarrow 3$ cloning experiment we have attained $\eta^{\text{expt}}(1,3) = 0.527 \pm 0.009$ in good agreement with the extrapolated value. The cloning protocol could be easily generalized, as said, by adopting any long BS chain according to the scheme of Fig. 1(a), and by a straightforward repetition of the previous procedures.

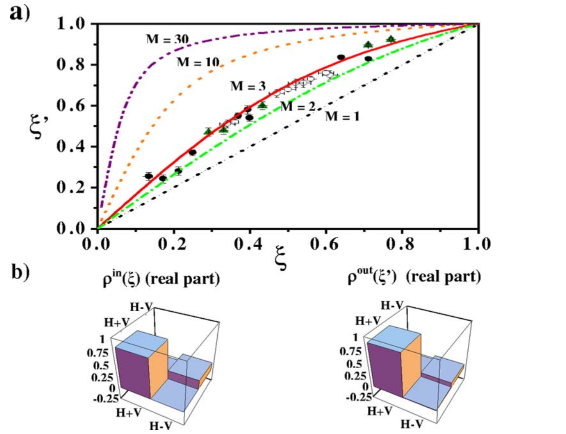

FIG. 3. (Color online) (a) The "degree of purity" ξ' realized after the application of the purification protocol, as a function of ξ, the input degree of purity. The experimental data correspond to the case $M=3$: the open circles, the filled circles, and the triangles denote, respectively, the $|H\rangle$, $|H+V\rangle$, and $|H+iV\rangle$ input states. The theoretical plots show the increase of the output purity for increasing M. (b) On the left is depicted the real part of the QST reconstructed density matrix of one input qubit for the case $\xi = 0.64$, drawn in its diagonal basis, while on the right is depicted the corresponding density matrix of one of the output purified qubits. The imaginary part contributions to the QST diagram have been found to be negligible.

Single-qubits purification. In addition to the above experiment, the symmetrization apparatus was applied to implement the qubit "purification" protocol proposed in [2] and recently realized for two qubits [7]. Generally, this protocol allows to distill, at the cost of the reduction of the process probability, a lesser number of purified qubits from $M \geqslant 2$ equally prepared, noisy input qubits. The output purity increases with M as shown by the theoretical curves in Fig. 3(a). The procedure can be adjusted to accomplish two different tasks: it can provide a high value of output fidelity with small probability or, alternatively, a lower gain of information with larger probability [2]. In this paper the first procedure has been selected and applied to $M=3$ equally prepared copies of the input state $\rho_{1,2,3}^{\mathrm{in}}=\xi|\phi\rangle\langle\phi|+\frac{1}{2}(1-\xi)I$ where $0\leqslant\xi\leqslant 1$. After successful SSP projection, all output qubits are found to be expressed by $\rho_{1,2,3}^{\mathrm{out}}=\xi'|\phi\rangle\langle\phi|+\frac{1}{2}(1-\xi')I$, i.e., affected by a larger purity factor: $\xi'=(5\xi+\xi^3)/(3+3\xi^2)\geqslant\xi$. This has been found to be the maximum achievable information gain obtainable by any probabilistic procedure [17]. In analogy with the previous experiments, the SSP operation was implemented by adopting the same two-step chain apparatus shown by Fig. 1(b). We have QST reconstructed the identical input and output qubits and measured the parameters ξ and ξ'. The protocol was carried out for different ξ-values and for three independent polarizations, $|H\rangle$, $|H+V\rangle$, and $|H+iV\rangle$, in order to show the universality of the process. Figure 3(a) reports the degree of purity ξ' achieved experimentally after the application of the purification protocol versus the input degree of purity ξ. Figure 3(b) shows the density QST pattern of a noisy qubit, in the diagonal basis, before and after the application of the SSP operation.

In conclusions we have reported a viable proposal of a very general and flexible multistep LO quantum symmetrization procedure. The scheme has been exploited to carry out the first successful realization of the $1\rightarrow 3$ UOQCM resulting from a concatenation of two unequal cloning steps: $(1\rightarrow 2)+(2\rightarrow 3)$. The fidelities of all these devices were found close to the theoretical limits. In addition, the same procedure has been tested by the realization of a $M=3$ universal qubit purification protocol, again with fidelities close to the theoretical limits. In perspective, by adoption of different postselection strategies, our BS-chain multistep method could act both as a reliable source of multiphoton entangled states (NOON states, W states, etc.) or as a Young-frame analyzer. Such device finds applications in several QI processes, such as the direct entanglement detection [18], the transmission of a reference frame [19], and the encoding of the QI in noiseless subspaces [20]. The present scheme can be further adopted to assemble phase covariant cloning machines from a UOQCM device [21]. Since the present method basically exploits the bosonic nature of the photons, it can be generally applied to any particle obeying Bose statistics.

This work has been supported by the QIPC FET European Network (Contract No. IST-2000-29681: ATESIT), by I.N.F.M. (PRA-CLON), and by M.I.U.R. (COFIN 2002).

[1] R. F. Werner, Phys. Rev. A **58**, 1827 (1998).
[2] J. I. Cirac, A. K. Ekert, and C. Macchiavello, Phys. Rev. Lett. **82**, 4344 (1999).
[3] F. Sciarrino, C. Sias, M. Ricci, and F. De Martini, Phys. Rev. A **70**, 052305 (2004).
[4] A. Barenco *et al.*, SIAM J. Comput. **26**, 1541 (1997).
[5] A. Peres, Int. J. Theor. Phys. **38**, 799 (1999).
[6] J. Ball, A. Dragan, and K. Banaszek, Phys. Rev. A **69**, 042324 (2004).
[7] M. Ricci *et al.*, Phys. Rev. Lett. **93**, 170501 (2004).
[8] M. Ricci, F. Sciarrino, C. Sias, and F. De Martini, Phys. Rev. Lett. **92**, 047901 (2004).
[9] W. T. M. Irvine *et al.*, Phys. Rev. Lett. **92**, 047902 (2004).
[10] K. Banaszek, A. Dragan, W. Wasilewski, and C. Radzewicz, Phys. Rev. Lett. **92**, 257901 (2004).
[11] M. Hendrych *et al.*, Phys. Lett. A **310**, 95 (2003).
[12] C. K. Hong, Z. Y. Ou, and L. Mandel, Phys. Rev. Lett. **59**, 2044 (1987).
[13] G. M. D'Ariano, C. Macchiavello, and P. Perinotti, e-print quant-ph/0506251.
[14] D. Bruß, J. Calsamiglia, and N. Lütkenhaus, Phys. Rev. A **63**, 042308 (2001).
[15] A. Lamas-Linares *et al.*, Science **296**, 712 (2002); F. De Martini *et al.*, Nature (London) **419**, 815 (2002); S. Fasel, N. Gisin, G. Ribordy, U. Scarani, and H. Zbinden, Phys. Rev. Lett. **89**, 107901 (2002); F. De Martini, D. Pelliccia, and F. Sciarrino, *ibid.* **92**, 067901 (2004).
[16] D. Bruß, A. Ekert, and C. Macchiavello, Phys. Rev. Lett. **81**, 2598 (1998).
[17] J. Fiurášek, Phys. Rev. A **70**, 032308 (2004).
[18] P. Horodecki and A. Ekert, Phys. Rev. Lett. **89**, 127902 (2002).
[19] G. Chiribella, G. M. D'Ariano, P. Perinotti, and M. F. Sacchi, Phys. Rev. Lett. **93**, 180503 (2004).
[20] L. Viola *et al.*, Science **293**, 5537 (2001).
[21] F. Sciarrino and F. De Martini, Phys. Rev. A (to be published).