Entanglement, EPR correlations, and mesoscopic quantum superposition by the high-gain quantum injected parametric amplification

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We investigate the multiparticle quantum superposition and the persistence of bipartite entanglement of the output field generated by the quantum injected high-gain optical parametric amplification of a single photon. The physical configuration based on the optimal universal quantum cloning has been adopted to investigate how the entanglement and the quantum coherence of the system persists for large values of the nonlinear parametric gain g.

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I. INTRODUCTION

In recent years, a large number of experiments aimed at the verification of fundamental aspects of quantum mechanics, such as quantum nonlocality, have been realized by adopting photon particles mutually interacting through nonlinear optical (NLO) processes. In addition sophisticated NLO methods have been extended to relevant investigations and realizations in the domain of the emerging new sciences of quantum information (QI) and quantum communication. In particular one such method, the quantum injected nonlinear (NL) parametric amplification (QIOPA) of single photon states, was particularly fruitful since it was adopted to provide the first experimental realization of the quantum cloning transformation, a fundamental QI concept [1–3]. This one provides the optimal distribution of the information contained in a quantum system, e.g., an N quantum-bit state, or *qubit*, onto a system of higher dimension, $M>N$. By virtue of the isomorphism existing between any logic qubit associated with spin 1/2 and the polarization state of a single photon, there it is generally supposed that N photons, identically prepared in an arbitrary state of polarization ($|\phi\rangle$), are injected into the amplifier on the input mode $\mathbf{k}_1$ [4,5]. The amplifier then generates on the output cloning mode (C) $M>N$ copies, or *clones* of the input qubit $|\phi\rangle$. Moreover, in the case of mode-nondegenerate QIOPA, the device simultaneously generates $M-N$ states $|\phi^{\perp}\rangle$ on the output *anticloning* mode $\mathbf{k}_2$ (ac) thus realizing a universal quantum NOT gate [6].

In the last years, the QIOPA scheme has been at the basis of experimental realizations of the $1\rightarrow 2$ universal optimal quantum cloning machine (UOQCM) [3,6–9] and of the $1\rightarrow 3$ phase covariant quantum cloning machine (PQCM) [10]. These tests, carried out in low power linearized conditions, i.e., with very low values of the NL parametric gain parameter $g\ll 1$, were followed recently by a series of OPA works, carried out in absence and in presence of quantum injection, which realized the high-gain (HG) spontaneous and stimulated generation of a large number of output photons M [11–13]. Within this new $N\rightarrow M$ cloning endeavor, a multiphoton superposition entangled state was generated, indeed a Schrödinger Cat state [14,15]. By virtue of the information-preserving (i.e., coherence-preserving) property of the parametric process, this implied the deterministic transferral of the well accessible and easily achievable quantum superposition condition affecting any input single-particle qubit to a "mesoscopic," i.e., multiparticle, amplified quantum state [12].

In the present article, we investigate theoretically the nature of quantum injected optical parametric amplification in high-gain regime. In particular we intend to investigate the most important property of the process, i.e., the entanglement of the output modes in the *multiparticle* condition. We show how this task can be undertaken by application of a technique, here referred to as "pair extraction technique," adopted to investigate the multiphoton states generated by high-gain spontaneous parametric down conversion (SPDC) [11,13].

In case of a bipartite entanglement, e.g., established over the output cloning and anticloning modes $\mathbf{k}_1$ and $\mathbf{k}_2$, the basic idea is to investigate a two-photon field component "extracted" out of the output multiphoton field and then infer the entanglement properties of the original field. Conceptually, the basic argument underlying this method consists of the impossibility of creating or enhancing the entanglement by any local operation, e.g., in this case by induced losses [16]. In order to do that, the adopted theoretical model takes into account the propagation losses leading to the imperfect detection of the output field. In the SPDC case, the explicit form for the two-photon output state has been found to exhibit a Werner state structure, i.e., consisting of a weighted mixture of a maximally entangled (singlet) state with a fully depolarized state [17]. This structure is resilient to losses for any value of the nonlinear gain parameter g. A similar approach will be applied to investigate theoretically the bipartite entanglement in the high-gain QIOPA process. The theoretical model enables one to obtain the explicit form of the two-photon output density matrix for any value of g. More precisely, the application of the method goes as follows. Let us start from a polarization entangled pair of photons associated with the output modes $\mathbf{k}_1$ and $\mathbf{k}_T$: Fig. 1. The photon created over $\mathbf{k}_T$ freely propagates and is detected by the phototube D_T while the photon associated to mode $\mathbf{k}_1$ is injected into the optical parametric amplifier. As a result of the

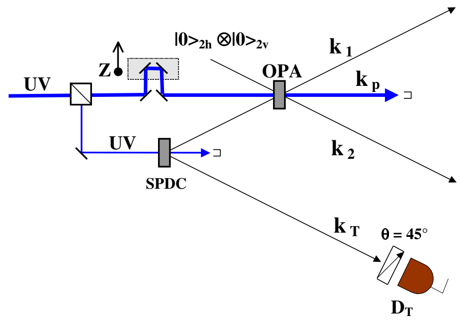

FIG. 1. (Color online) Schematic diagram of the single photon *quantum injected* optical parametric amplifier (QIOPA). The injection is provided by an external spontaneous parametric down conversion source of polarization entangled photon states [4]. The losses are simulated by the insertion of a beam splitter (BS) over each propagation mode $\mathbf{k}_i$.

coherence-preserving nature of the amplification process the quantum correlations between the initial two photon entangled states are transferred into correlations between the photon on mode $\mathbf{k}_T$ (hereafter referred to as the "trigger") and the output field of the QIOPA device.

In order to characterize the field generated over the three output modes $\mathbf{k}_T$, $\mathbf{k}_1$, and $\mathbf{k}_2$ in the high-gain regime, we explicitly derive the reduced density matrix of the three photon states obtained over all the output modes adopting the pair extraction technique. Hence we shall investigate the characteristics of each bipartite system: trigger-cloning, trigger-anticloning, and cloning-anticloning modes. Each subsystem has been found to exhibit a Werner structure. By adopting this model we are able to analyze the general entanglement properties of the overall three mode system. Furthermore, this leads to the demonstration of the persistence of entanglement for the two systems trigger-cloning and cloning-anticloning for any large value of the gain parameter g, in our opinion the main result of the present work [16].

The article is structured as follows. In Sec. II we present the method to extract from multiphoton states generated by the amplification process two photon components making use of lossy channel and imperfect detection. In Sec. III we analyze the quantum correlations between trigger, cloning, and anticloning, realizing a model for the three qubit density matrix with the same projection scheme. Finally, Sec. IV is devoted to the discussions of agreement with the experimental apparatus and the perspectives of the present research.

II. QIOPA IN THE PAIR-EXTRACTION REGIME

In this section, we analyze the amplified multiphoton quantum injected state with particular emphasis on the effect of losses on bipartite correlations. Let us consider the experimental scheme reported in Fig. 1. A single photon (mode $\mathbf{k}_1$) is injected into the nonlinear (NL) crystal, typically a BBO (β-barium-borate), cut for type-II phase matching and excited by a sequence of uv (ultraviolet) mode-locked laser pulses of wavelength (wl) λ_p propagating along the mode $\mathbf{k}_P$. The relevant modes of the NL three-wave interaction driven by the uv pulses associated with mode $\mathbf{k}_p$ are the two spatial modes with wave vector (wv) $\mathbf{k}_i$, $i=1,2$, each one supporting the two horizontal (H) and vertical (V) polarizations of the interacting photons. The QIOPA is λ degenerate, i.e., the interacting photons have the same wl's $\lambda=2\lambda_p$. The injected single photon is provided by an external spontaneous parametric down conversion source of biphoton states [4].

The Hamiltonian of the parametric down-conversion process in the interaction picture reads [3,4]

$$\hat{H}=i\kappa(\hat{a}^{\dagger}_{1H}\hat{a}^{\dagger}_{2V}-\hat{a}^{\dagger}_{1V}\hat{a}^{\dagger}_{2H})+\text{H.c.},\tag{1}$$

where $\hat{a}^{\dagger}_{ij}$ represents the creation operators associated to the spatial propagation mode $\mathbf{k}_i$, with polarization $j=\{H,V\}$. The NL coupling constant κ depends on the nonlinearity of the crystal and is proportional to the amplitude of the pump beam. It has been theoretically shown [18,19] that $\hat{H}$ is invariant under simultaneous general SU(2) transformations of the polarization vectors for modes $\mathbf{k}_1$ and $\mathbf{k}_2$. We may, then, cast the above expression (1) in the following form:

$$\hat{H}=i\kappa(\hat{a}^{\dagger}_{\Psi}\hat{b}^{\dagger}_{\Psi\perp}-\hat{a}^{\dagger}_{\Psi\perp}\hat{b}^{\dagger}_{\Psi})+\text{H.c.},\tag{2}$$

where the field labels refer to two mutually orthogonal polarization unit vectors for each mode, Ψ and $\Psi^{\perp}$, corresponding respectively to the state vectors: $|\Psi\rangle$ and $|\Psi^{\perp}\rangle$. This Hamiltonian generates a unitary transformation $\hat{U}=e^{-i\hat{H}t/\hbar}$ acting on the input multimode photon state.

Let us consider first the injection of a polarization encoded *qubit* in the QIOPA system:

$$|\Psi\rangle_{IN}\equiv\alpha|\Psi\rangle^{H}_{IN}+\beta|\Psi\rangle^{V}_{IN}\tag{3}$$

with $|\alpha|^2+|\beta|^2=1$, defined in the (2×2)-dimensional Hilbert space of polarizations $(\vec{\pi})$. We analyze this configuration by accounting for the two interacting optical modes $\mathbf{k}_1$ and $\mathbf{k}_2$, i.e., in terms of the basis vectors: $|\Psi\rangle^{H}_{IN}=|1\rangle_{1H}|0\rangle_{1V}|0\rangle_{2H}|0\rangle_{2V}\equiv|1,0,0,0\rangle$, $|\Psi\rangle^{V}_{IN}=|0,1,0,0\rangle$. In virtue of the general *information preserving* property of any nonlinear (NL) transformation of parametric type, the output state is again expressed by a "multiparticle qubit":

$$|\Psi\rangle\equiv\hat{U}|\Psi\rangle_{IN}=\hat{U}(\alpha|\Psi\rangle^{H}_{IN}+\beta|\Psi\rangle^{V}_{IN})=\alpha|\Psi\rangle^{H}+\beta|\Psi\rangle^{V}.\tag{4}$$

Since $|\Psi\rangle_{IN}$ is a *pure* state and $\hat{U}$ is *unitary*, $|\Psi\rangle$ is also a pure state. Furthermore and most important, it consists of a quantum superposition of two orthonormal, multiparticle states $|\Psi\rangle^{H}$ and $|\Psi\rangle^{V}$.

$$|\Psi\rangle^{H}\equiv\gamma\sum_{i,j=0}^{\infty}(-\Gamma)^{i+j}(-1)^{j}\sqrt{i+1}|i+1,j,j,i\rangle,\tag{5}$$

$$|\Psi\rangle^{V}\equiv\gamma\sum_{i,j=0}^{\infty}(-\Gamma)^{i+j}(-1)^{j}\sqrt{j+1}|i,j+1,j,i\rangle,\tag{6}$$

where $C\equiv\cosh g$, $\gamma\equiv C^{-3}$, $\Gamma\equiv\tanh g$, and $g\equiv\kappa\overline{t}_{int}$ expresses the NL gain of the parametric process, $\overline{t}_{int}$ being the interaction time.

In order to implement the *pair extraction technique* described above in Sec. I, we may now explicitly derive the theoretical expression of the "pair extracted" density matrix in the regime of photon losses on all QIOPA spatial and polarization output modes. According to a standard procedure, the losses are simulated by the insertion of dummy ideal beam splitters (BS) on the optical modes, followed by ideal detectors [20]. Let us first express the above QIOPA output wave function in absence of losses, in terms of vacuum states:

$$|\Psi\rangle = \gamma\sum_{i,j=0}^{\infty}(-\Gamma)^{i+j}\frac{(-1)^j}{j\,!\,i!}(\hat{a}_{1H}^{\dagger(i+1)}\hat{a}_{1V}^{\dagger j}\hat{a}_{2H}^{\dagger j}\hat{a}_{2V}^{\dagger i} - \hat{a}_{1H}^{\dagger i}\hat{a}_{1V}^{\dagger(j+1)}\hat{a}_{2H}^{\dagger j}\hat{a}_{2V}^{\dagger i})|0,0,0,0\rangle. \quad (7)$$

After insertion of the BS, the lossless QIOPA output field operators $\{\hat{a}_{ij}^{\dagger}\}$ are transformed into the $\{\hat{a}_{ij-OUT}^{\dagger}\}$ acting again on the output lossy channels, by the unitary BS map:

$$\begin{pmatrix}\hat{a}_{ij-OUT}^{\dagger}(t)\\ \hat{b}_{ij-OUT}^{\dagger}(t)\end{pmatrix} = \begin{pmatrix}\sqrt{\eta} & \sqrt{1-\eta}\\ \sqrt{1-\eta} & \sqrt{\eta}\end{pmatrix}\begin{pmatrix}\hat{a}_{ij}^{\dagger}(t)\\ \hat{b}_{ij}^{\dagger}(t)\end{pmatrix}. \quad (8)$$

Here the operators $\{\hat{b}_{ij}^{\dagger}\}$ and $\{\hat{b}_{ij-OUT}^{\dagger}\}$ correspond to the input and output side, i.e., "reflected modes" of the BS that do not coincide with the QIOPA states. Precisely, the set $\{\hat{b}_{ij}^{\dagger}\}$ and $\{\hat{b}_{ij-OUT}^{\dagger}\}$ act correspondingly on a set of side vacuum states and on the set of "reflected modes" $\{\widetilde{\mathbf{k}}_i^{OUT}\}$. We assume further that all BS are equal and characterized by a common spatially and polarization independent transmittivity parameter η.

The lossy effect of the BS maps $|\Psi\rangle$ into an output state $|\Psi\rangle_{OUT}^{BS}$ which involves the four output "transmitted modes" and the four output "reflected modes" of the two BS. Precisely, $|\Psi\rangle_{OUT}^{BS}$ is expressed in terms of Fock states: $|n_{1H},n_{1V},n_{2H},n_{2V}\rangle_a\otimes|n_{1H},n_{1V},n_{2H},n_{2V}\rangle_b$ where the two terms in the tensor product represent, respectively, the output transmitted BS modes ($\hat{a}$ modes) and the output reflected modes ($\hat{b}$ modes). The output state of the overall {QIOPA +BS} system is found to reproduce the quantum superposition behavior

$$|\Psi\rangle_{OUT}^{BS} = \gamma\sum_{i,j=0}^{\infty}\frac{(-\Gamma)^{i+j}(-1)^j}{i\,!\,j!}(-\jmath\sqrt{1-\eta^2})^{2*(i+j)+1} \times(\alpha|\Phi_{i,j}^{H}\rangle + \beta|\Phi_{i,j}^{V}\rangle), \quad (9)$$

where

$$|\Phi_{i,j}^{H}\rangle = \sum_{l_1=0}^{i+1}\sum_{l_2=0}^{j}\sum_{l_3=0}^{j}\sum_{l_4=0}^{i}\left(\frac{\eta}{-\jmath\sqrt{1-\eta^2}}\right)^{(l_1+l_2+l_3+l_4)} \times j\,!\sqrt{\binom{i+1}{l_1}\binom{j}{l_2}\binom{j}{l_3}\binom{i}{l_4}(i+1)\,!\,i!} \times|l_1,l_2,l_3,l_4\rangle_a|i+1-l_1,j-l_2,j-l_3,i-l_4\rangle_b \quad (10)$$

$$|\Phi_{i,j}^{V}\rangle = \sum_{l_1=0}^{i}\sum_{l_2=0}^{j+1}\sum_{l_3=0}^{j}\sum_{l_4=0}^{i}\left(\frac{\eta}{-\jmath\sqrt{1-\eta^2}}\right)^{(l_1+l_2+l_3+l_4)} \times i\,!\sqrt{\binom{i}{l_1}\binom{j+1}{l_2}\binom{j}{l_3}\binom{i}{l_4}(j+1)\,!\,j!} \times|l_1,l_2,l_3,l_4\rangle_a|i-l_1,j+1-l_2,j-l_3,i-l_4\rangle_b, \quad (11)$$

with $\jmath^2=-1$. All undetected BS "reflected modes" must then be discarded, i.e., traced out in the above expressions leading to the *reduced* density matrix $\rho'=\mathrm{Tr}_b[|\Psi\rangle_{OUT}^{BS}\,{}_{OUT}^{BS}\langle\Psi|]$ over the transmitted modes $\{\mathbf{k}_i^{OUT}\}$ that can be expressed as follows:

$$\rho' = \sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\sum_{k_3=0}^{\infty}\sum_{k_4=0}^{\infty}\langle k_1,k_2,k_3,k_4|_b|\Psi\rangle_{OUT}^{BS}\langle\Psi||k_1,k_2,k_3,k_4\rangle_b. \quad (12)$$

Up to now we have considered arbitrary, polarization-symmetric channel losses. In the following we make the additional assumption of high losses, which will greatly simplify the calculations. The explicit expression of ρ' may now be easily obtained in the high-loss regime, i.e., identified by the relation $\eta\bar{n}\ll1$, $\eta\bar{n}$ being the average number of photons transmitted by the BS per mode. By virtue of this condition we may drop the terms of the sums proportional to η^n for $n>2$. This corresponds to consider only matrix elements of a representation in which any Fock basis state corresponds to a photon occupation number $n\leq2$. As final step we assume to detect one photon on each mode $\mathbf{k}_i^{OUT}$ by a standard two-photon coincidence technique over the QIOPA output modes, $\mathbf{k}_1$ and $\mathbf{k}_2$. By this technique we only investigate output states involving one photon emitted over these two modes. This output condition is expressed by a matrix representation of ρ' involving the basis states $\{|1,0,1,0\rangle,|1,0,0,1\rangle,|0,1,1,0\rangle,|0,1,0,1\rangle\}$, corresponding to the basis $\{|H\rangle_1|H\rangle_2,|H\rangle_1|V\rangle_2,|V\rangle_1|H\rangle_2,|V\rangle_1|V\rangle_2\}$. The explicit expression of the detected "pair extracted" density matrix is finally given by the expression

$$\rho' = \frac{1}{3d}\begin{pmatrix}|\alpha|^2d+|\beta|^22t^2 & \beta\alpha^*d & -\beta\alpha^*2 & 0\\ \alpha\beta^*d & |\alpha|^22(1+d)+|\beta|^2d & -|\alpha|^22-|\beta|^22 & \beta\alpha^*2\\ -\alpha\beta^*2 & -|\alpha|^22-|\beta|^22 & |\alpha|^2d+|\beta|^22(1+d) & -\beta\alpha^*d\\ 0 & \alpha\beta^*2 & -\alpha\beta^*d & |\alpha|^22t^2+|\beta|^2d\end{pmatrix} \quad (13)$$

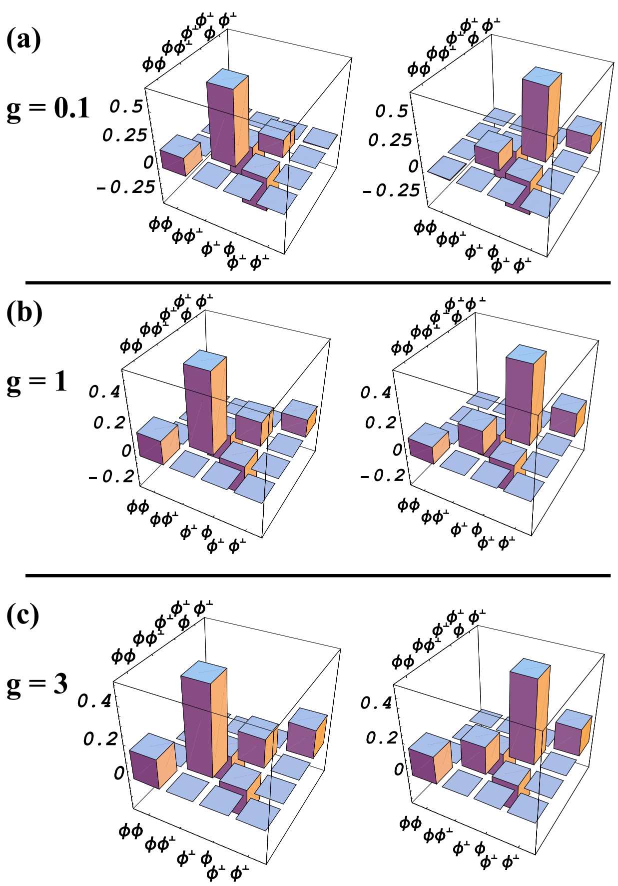

FIG. 2. (Color online) Theoretical density matrix of the reduced two-photon output state over the output modes $\mathbf{k}_i$ $\{i=1,2\}$ conditioned by the injection of a generic input polarization qubit $|\phi\rangle$ (left column) and $|\phi^{\perp}\rangle$ (right column) on the mode $\mathbf{k}_1$. (a) corresponds to $g=0.1$, (b) to $g=1$, (c) to $g=3$. The imaginary parts with all matrix elements equal to zero are not reported.

with $d=(1+t^2)$ and $t=\Gamma(1-\eta^2)$. Let us consider the particular asymptotic case $\eta \to 0$, i.e., $t \simeq \tanh g$. Figure 2 refers to different injection states and to different values of the interaction parameter g. There the tomographic patterns are reproduced identically for any couple of different orthogonal input states $\{|\phi\rangle, |\phi^{\perp}\rangle\}$ as a consequence of the universality of this process, Eq. (2). Furthermore, the structure of the 4×4 matrices shows the relevant quantum features of the output state. For instance, the highest peak on the diagonals expressing the quantum superposition of the input state shifts from the position $|\phi\phi^{\perp}\rangle\langle\phi\phi^{\perp}|$ to $|\phi^{\perp}\phi\rangle\langle\phi^{\perp}\phi|$ in correspondence with the OPA excitation by any set of generic orthogonal injection states $\{|\phi\rangle, |\phi^{\perp}\rangle\}$.

Let us estimate the entanglement of the *pair extracted* output state (13). Owing to the tested universality of the amplification process we restrict the present analysis to the input qubit $|\Psi\rangle_{IN} \equiv |H\rangle$, i.e., $\alpha=1$, $\beta=0$. In this case the two-photon density matrix reads

$$\rho' = \frac{1}{3(1+t^2)} \begin{pmatrix} \frac{1}{2}(1+t^2) & 0 & 0 & 0 \\ 0 & (2+t^2) & -1 & 0 \\ 0 & -1 & \frac{1}{2}(1+t^2) & 0 \\ 0 & 0 & 0 & t^2 \end{pmatrix}. \tag{14}$$

The entanglement measurement, the "concurrence" is $C(\rho') = [2/3(1+t^2)](1-\frac{t}{2}\sqrt{1+t^2})$ [21]. We found $C(\rho')>0$ for any value of g and $C(\rho')\to 0$ for $g\to\infty$. This result complies with the one obtained in the case of the $1\to M$ universal quantum cloning by Ref. [22]. There it was shown that in this cloning process closely parallel to the present configuration, the concurrence between a clone and an ancilla is always different from zero for any value of M and vanishes in the limit $M\to\infty$.

We now analyze the output state (14) over each one of the output modes, by further tracing ρ'. For the mode $\mathbf{k}_1$ we obtain

$$\rho'_{k1} = \frac{1}{6(1+t^2)} \begin{pmatrix} 5+3t^2 & 0 \\ 0 & 1+3t^2 \end{pmatrix}. \tag{15}$$

The fidelity between the output state and the input is $F(|H\rangle, \rho'_{k1}) = \langle H|\rho'_{k1}|H\rangle = (5+3t^2)/6(1+t^2)$. In the limit $g\to\infty$ we obtain $F=\frac{2}{3}$ while the limit $g\to 0$ leads to $F=\frac{5}{6}$. In the present investigation, dealing with the generation of a pair of photons, we retrieved exactly the result obtained for the $1\to 2$ optimal cloning process. The average number of photons over the mode $\mathbf{k}_1$ is equal to $3\bar{n}+1$ with $\bar{n}=\sinh^2 g$. Hereafter the mode $\mathbf{k}_1$ will also be referred to as the *cloning mode*.

The single photon polarization state over mode $\mathbf{k}_2$ is described by the density matrix for any g value:

$$\rho'_{k2} = \begin{pmatrix} 1/3 & 0 \\ 0 & 2/3 \end{pmatrix}. \tag{16}$$

The fidelity between the output state and the orthogonal of the input one $F(|V\rangle, \rho'_{k2}) = 2/3$. Hereafter the mode $\mathbf{k}_2$ will be referred to as the *anticloning mode*.

III. SCHRÖDINGER CAT STATE: PERSISTENCE OF ENTANGLEMENT

Let us now consider a different, more complex configuration in which the injected photon belongs to a polarization entangled pair: see Fig. 3. The previous theoretical method will be adopted here to investigate the properties of the overall output field. Let us now go into details. The initial pump beam is split by an unbalanced beam splitter (BS_P) into two beams. A low intensity beam $\mathbf{k}'_P$ excites the NL crystal A which generates pairs of entangled photons over the modes $\mathbf{k}_1$ and $\mathbf{k}_T$:

$$|\Phi^-\rangle_{kT,k1} = 2^{-1/2}(|H\rangle_{kT}|H\rangle_{k1} - |V\rangle_{kT}|V\rangle_{k1}), \tag{17}$$

where subscripts $\mathbf{k}_T$, $\mathbf{k}_1$ refer to trigger and injection photons, propagating along $\mathbf{k}_T$ and $\mathbf{k}_1$ modes, respectively. The pump

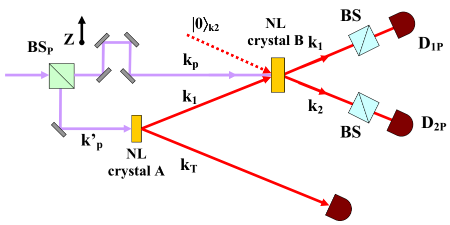

FIG. 3. (Color online) Schematic diagram of the *quantum injected* optical parametric amplifier (QIOPA). The pump beam $\mathbf{k}'_P$ excites the NL crystal A which generates pairs of entangled photons over the modes $\mathbf{k}_1$ and $\mathbf{k}_T$. The photon over the mode $\mathbf{k}_T$ provides the trigger of the overall experiment while the twin photon over the mode $\mathbf{k}_1$ excites the NL crystal B together with the pump beam $\mathbf{k}_P$ [4]. The losses are simulated by the insertion of a beam splitter (BS) over each propagation mode $\mathbf{k}_i$.

power is sufficiently low to avoid the simultaneous generation of more than one pair of photons. The photon over the mode $\mathbf{k}_T$ provides the trigger of the overall experiment while the twin photon over the mode $\mathbf{k}_1$ excites the NL crystal B together with the pump beam $\mathbf{k}_P$ [4].

The overall dynamic of the process is described by the unitary operator $\hat{V}=\hat{I}_{kT}\otimes\hat{U}_{k1}$ acting on the initial state $|\Phi^-\rangle_{Tk1}\otimes|0\rangle_{k2}$, where $\hat{U}_{k1}$ is the time evolution operator acting on the injection state and $\hat{I}_{kT}$ is the unit matrix acting on the trigger state. Using the results of the previous section we obtain

$$|\Sigma\rangle = \hat{V}|\Phi^-\rangle_{kT,k1} \tag{18}$$

$$=2^{-1/2}\gamma\{|H\rangle_{kT}\otimes\sum_{i,j=0}^{\infty}(-\Gamma)^{i+j}(-1)^j\sqrt{i+1}|i+1,j,j,i\rangle \tag{19}$$

$$-|V\rangle_{kT}\otimes\sum_{i,j=0}^{\infty}(-\Gamma)^{i+j}(-1)^j\sqrt{j+1}|i,j+1,j,i\rangle. \tag{20}$$

This multiparticle quantum state exhibits an entangled structure connecting the microscopic property of the system, i.e., the single particle "trigger" acting on the mode $\mathbf{k}_T$, and the macroscopic quantum superposition. This indeed corresponds to the original definition of "Schrödinger Cat state" [14,15].

In the following we apply the pair extraction method to analyze the entanglement properties of the overall wave function (19). High losses are introduced over the modes $\mathbf{k}_i$ $\{i=1.2\}$. Detection of photons over the three modes $\mathbf{k}_i$, $\mathbf{k}_T$ enables us to reconstruct the density matrix through a tomographic technique. The theoretical model shows the presence of entanglement between the trigger photon and each one of the amplified photons. Since the entanglement is not ascribable to the subsequent amplification process which is acting locally on the $\mathbf{k}_1$ arm of the pair, one is forced to conclude that the original trigger-injection entanglement has survived the QIOPA amplification.

Under the *pair extraction* approximation on $\mathbf{k}_i$ $\{i=1.2\}$, i.e., after applying high losses on the output modes before detection, the expression of the overall normalized three qubit pair extracted density matrix is easily found:

$$\rho'' = \begin{pmatrix} \frac{1}{12} & 0 & 0 & 0 & 0 & \frac{1}{12} & -\frac{1}{6+6t^2} & 0 \\ 0 & \frac{2+t^2}{6+6t^2} & -\frac{1}{6+6t^2} & 0 & 0 & 0 & 0 & -\frac{1}{6+6t^2} \\ 0 & -\frac{1}{6+6t^2} & \frac{1}{12} & 0 & 0 & 0 & 0 & \frac{1}{12} \\ 0 & 0 & 0 & \frac{t^2}{6+6t^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{t^2}{6+6t^2} & 0 & 0 & 0 \\ \frac{1}{12} & 0 & 0 & 0 & 0 & \frac{1}{12} & -\frac{1}{6+6t^2} & 0 \\ -\frac{1}{6+6t^2} & 0 & 0 & 0 & 0 & -\frac{1}{6+6t^2} & \frac{2+t^2}{6+6t^2} & 0 \\ 0 & -\frac{1}{6+6t^2} & \frac{1}{12} & 0 & 0 & 0 & 0 & \frac{1}{12} \end{pmatrix}. \tag{21}$$

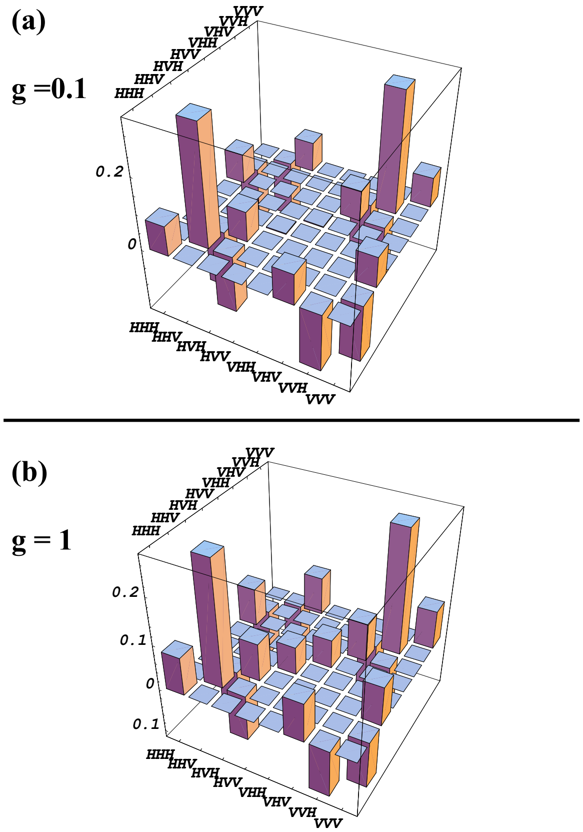

FIG. 4. (Color online) Theoretical density matrix of the reduced three-photon output state over the modes $\mathbf{k}_T$, $\mathbf{k}_i$ $\{i=1,2\}$. (a) corresponds to $g=0.1$, (b) to $g=1$. The imaginary parts with all matrix elements equal to 0 are not reported.

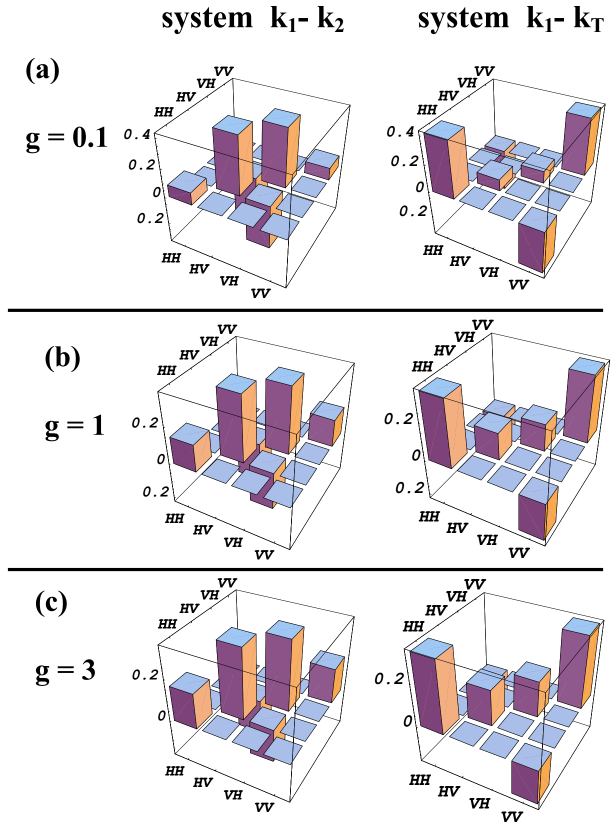

FIG. 5. (Color online) Theoretical density matrix of the reduced two-photon output state over the output modes $\mathbf{k}_1$, $\mathbf{k}_2$ (left column) and $\mathbf{k}_1$, $\mathbf{k}_T$ (right column). (a) corresponds to $g=0.1$, (b) to $g=1$, (c) to $g=3$. The imaginary parts with all matrix elements equal to 0 are not reported.

Figure 4 reports density matrices for two different values of interaction parameter g.

In order to understand at a deeper level the multiparticle quantum cloning process, we may consider the (4×4)-dimensional matrices obtained by tracing ρ'' over the remaining state of the manifold $\{\mathbf{k}_i,\mathbf{k}_T\}$.

Let us start from the system consisting of the $\mathbf{k}_i$ $\{i=1.2\}$ modes, i.e., by the cloning-anticloning systems. The reduced density matrix $\rho''_{k1,k2}=\mathrm{Tr}_{kT}(\rho'')$ of the extracted two photons is found,

$$\rho''_{k1,k2}=\begin{pmatrix} \frac{1-p}{4} & 0 & 0 & 0 \\ 0 & \frac{1+p}{4} & -\frac{p}{2} & 0 \\ 0 & -\frac{p}{2} & \frac{1+p}{4} & 0 \\ 0 & 0 & 0 & \frac{1-p}{4} \end{pmatrix}, \tag{22}$$

with $p=\frac{2}{3}\frac{1}{1+t^2}$. We note that the above density matrix has the form of a Werner state (WS) $\rho_W=p|\Psi_-\rangle\langle\Psi_-|+\frac{1-p}{4}I$, i.e., a mixture of the maximally entangled state $|\Psi_-\rangle_{1,2}=2^{-1/2}(|H\rangle_1|V\rangle_2-|V\rangle_1|H\rangle_2)$ with probability p and of the fully chaotic state $I/4$, being I the identity operator on the overall Hilbert space. Note that for any finite value of the NL value g, the singlet probability is $p>\frac{1}{3}$ reaching asymptotically the value $p=\frac{1}{3}$ for $g\rightarrow\infty$. Since the condition $p>\frac{1}{3}$ is a necessary and sufficient one for state nonseparability of any WS [23], the entanglement condition affecting the pair extracted $\rho''_{k1,k2}$, and then of its multiparticle counterpart $\rho_{k1,k2}$, *does persist* for any finite value of g. All this can be compared with a similar result obtained in the regime of spontaneous parametric down conversion, i.e., with no injection of a single photon state (see Fig. 5). In the SPDC case it was found $\bar{p}=\frac{1}{1+2t^2}$ [13]. The concurrence of the state (22) is $C_{k1,k2}=2\max\left(\frac{3p-1}{4},0\right)=\frac{1}{2}\left(\frac{1-t^2}{1+t^2}\right)=\frac{1}{2}(\sinh^2 g+\cos^2 g)^{-1}$. In case of high-gain value we find $C_{k1,k2}\simeq\frac{1}{\bar{n}}$ and $\tau_{kT,k1}=\frac{1}{C^2_{T,k1}}=\frac{1}{\bar{n}^2}$. The average number of output qubits for each mode is equal to $\simeq 3\bar{n}$.

A similar analysis can be carried out for the correlation affecting the modes $\mathbf{k}_T$ and $\mathbf{k}_1$, as shown by the structure of the pair extracted $\rho''_{kT,k1}\equiv\mathrm{Tr}_{k2}(\rho'')$,

$$\rho''_{kT,k1} = \begin{pmatrix} \frac{1+q}{4} & 0 & 0 & -\frac{q}{2} \\ 0 & \frac{1-q}{4} & 0 & 0 \\ 0 & 0 & \frac{1-q}{4} & 0 \\ -\frac{q}{2} & 0 & 0 & \frac{1+q}{4} \end{pmatrix}, \quad (23)$$

with $q=\frac{2}{3}\frac{1}{1+t^2}$. Note that once again the density matrix bears the WS structure $\rho=q|\Phi_-\rangle\langle\Phi_-|+\frac{1-q}{4}I$, i.e., is a mixture with probability q of the maximally entangled state $|\Phi_-\rangle_{1,2}=2^{-1/2}(|H\rangle_1|H\rangle_2-|V\rangle_1|V\rangle_2)$ and of the maximally chaotic state $I/4$. Once again the entanglement condition $q>\frac{1}{3}$ is met for any finite value of g and $q\rightarrow\frac{1}{3}$ for $g\rightarrow\infty$. Note also that in the limit of small $g\approx 0$ is $q\approx\frac{2}{3}$ (the present approach considers at least the generation of a pair of photons). The concurrence of the state of Eq. (23) is found, $C_{kT,k1}=\frac{1}{2}(\sinh^2 g+\cosh^2 g)^{-1}$. In case of large gain value is found, $C_{kT,k1}\simeq\frac{1}{\bar{n}}$ and $\tau_{kT,k1}=C^2_{kT,k1}=\frac{1}{\bar{n}^2}$. The average number of output clones is equal to $3\bar{n}+1\simeq 3\bar{n}$.

Consider at last the pair extracted reduced density matrix involving the $\mathbf{k}_2$ and $\mathbf{k}_T$ modes, $\rho''_{kT,k2}\equiv\mathrm{Tr}_{k1}(\rho'')$. Its expression is found to be gain independent,

$$\rho''_{kT,k2} = \begin{pmatrix} \frac{1}{6} & 0 & 0 & \frac{1}{6} \\ 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{6} \end{pmatrix}. \quad (24)$$

It also exhibits a WS structure, $\rho''_{kT,k2}=l|\Phi_+\rangle\langle\Phi_+|+\frac{1-l}{4}I$ with $l=\frac{1}{3}$ and $|\Phi_+\rangle_{1,2}=2^{-1/2}(|H\rangle_1|H\rangle_2+|V\rangle_1|V\rangle_2)$. As a consequence, the above state is *separable*, a result in agreement with the theory of the optimal quantum machines [24]. Indeed the QIOPA has been shown to implement over the mode $\mathbf{k}_2$ the universal optimal flipping machine [8]. The completely positive (CP) map which implements the UNOT (universal UNOT) gate has the following Kraus representation $\mathcal{E}_{\mathrm{UNOT}}(\rho)=\frac{1}{3}(\sigma_X\rho\sigma_X+\sigma_Y\rho\sigma_Y+\sigma_Z\rho\sigma_Z)$. The state $\rho''_{kT,k2}$ can be obtained applying to the entangled state $|\Phi^-\rangle_{kT,k1}$ the identity over the mode $\mathbf{k}_T$ and the CP map $\mathcal{E}_{\mathrm{UNOT}}$ over the mode $\mathbf{k}_1$,

$$I_{kT}\otimes\mathcal{E}_{\mathrm{UNOT}-k1}(|\Phi^-\rangle_{kT,k1}\langle\Phi^-|_{kT,k1}) = \frac{1}{3}\left[\begin{matrix}|\Phi^+\rangle_{kT,k2}\langle\Phi^+|_{kT,k2}+|\Psi^-\rangle_{kT,k2}\langle\Psi^-|_{kT,k2}\\ +|\Psi^+\rangle_{kT,k2}\langle\Psi^+|_{kT,k2}\end{matrix}\right]. \quad (25)$$

Interestingly enough, the last expression can be shown to be equal to Eq. (24). This confirms the overall validity of the present approach.

IV. DISCUSSIONS AND CONCLUSIONS

In the present paper, the theory of quantum injected optical parametric amplification has been extensively investigated in regime of high gain and high losses with particular attention for the entanglement properties of the output fields. We have exploited the developed theoretical tool to investigate the properties of entangled states after a cloning process, demonstrating the persistence of entanglement for any clone-trigger subsystem. Connections with Werner state have been established in the physical process of stimulated emission. In addition, we have shown that the properties of QIOPA output state do comply with the ones conventionally expected for a Schrödinger Cat system [14,15]. With respect to the persistence of the coherence of the latter system for increasing "size," an interesting problem could be the investigation on the entanglement persistence for increasing the value of the gain g, i.e., for a very large number of generated clones. Unfortunately, we expect that for an increasing number of clones, the small amount of bipartite entanglement should not be observable due to experimental imperfections (such as walk off effects and other sources of decoherence). However the density matrices experimentally measured in Refs. [11–13] have been found to be in good agreement with the theoretical ones. On the other hand, the technique introduced above turns out to be a useful tool to investigate single photon features of mesoscopic fields.

An interesting aspect which deserves further investigation is the effect of losses on the orthogonality condition of the two wave functions of Eqs. (5) and (6), in particular how the detection efficiency influences the distinction of the two initial orthogonal terms. The two extreme condition can easily be derived. While for vanishing losses the Hilbert-Schimdt distance of the two interfering state is found to be $d(|\Psi\rangle^H,|\Psi\rangle^V)=\mathrm{Tr}[(|\Psi\rangle^H\langle\Psi|-|\Psi\rangle^V\langle\Psi|)^2]=2$, the condition of very low efficiency of detection leads to $d(\rho'^H_{k1},\rho'^V_{k1})=2/9$. Finally a new approach to investigate quantum features of the optical field based on the combination of data obtained with different detection efficiencies [25] could improve the present results.

The theoretical results obtained above have been carefully tested by the experiment reported in Ref. [12]. The adopted optical scheme was similar to the diagram shown in Fig. 3 but for more compact "folded" configurations by which a single NL crystal slab, excited in both directions by the UV "pump" laser beam, realized in sequence the SPDC and the QI-OPA operations. The experimental investigation of the multiphoton superposition and entanglement implied by Eqs. (14) and (21) was carried out by means of quantum state tomography (QST) according to the pair extraction method previously described. The beams associated with the output modes $\mathbf{k}_i$ $(i=1,2)$ were highly attenuated to the single-photon level by the two low transmittivity BS. Experimentally the maximal value of gain obtained has been found as $g_{\mathrm{exp}}=(1.19\pm0.05)$, while the detection quantum efficiencies read $\eta_1=(4.9\pm0.2)\%$ and $\eta_2=(4.2\pm0.2)\%$. By the previous values we found the condition $\eta\bar{n}\simeq 0.1$. The QST analysis of the *reduced* output state $\rho''_{k1,k2}$ determined by the set of input $|\Psi\rangle_{in}$: $\{|H\rangle,|V\rangle,|\pm\rangle\}$. The good agreement between theory $\rho''_{k1,k2}$ and experiment

$\rho''^{\text{exp}}_{k1,k2}$ is expressed by the measured average Uhlmann "fidelity":

$$\mathcal{F}(\rho''^{\text{exp}}_{k1,k2},\rho''_{k1,k2}) \equiv [\mathrm{Tr}(\sqrt{\rho''^{\text{exp}}_{k1,k2}}\rho''_{k1,k2}\sqrt{\rho''^{\text{exp}}_{k1,k2}})^{1/2}]^2$$
$$= (96.6 \pm 1.2)\% .$$

Finally, the QST reconstruction of the density matrix ρ''^{exp} has been carried out with a fidelity $\mathcal{F}(\rho''^{\text{exp}},\rho'')$ $=(85.0\pm 1.1)\%$.

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