Generation and Characterization of Werner States and Maximally Entangled Mixed States by a Universal Source of Entanglement

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We present a novel technique for generating two-photon polarization mixed states of any structure, which is based on the peculiar spatial characteristics of a high brilliance source of entangled pairs. Werner states and maximally entangled mixed states, two well-known families of mixed states important for quantum information, have been created and fully characterized by this technique. We have also investigated and tested the nonlocal properties of these states.

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In modern science of quantum information (QI), entanglement represents the basis underlying future quantum computers [1], quantum teleportation [2,3], and some kinds of cryptographic communications [4]. The quest for the pure maximally entangled states is somewhat counterbalanced by the increasing emergence of the mixed states and nonmaximally entangled states in the real world. Because of the unavoidable effects of losses and decohering interactions, indeed these states are today considered the basic constituents of modern QI and computation as they determine the performance of all quantum communication protocols [2,3,5]. In the past few years, within an investigation aimed at the characterization of the mixed states as a practical resource, a new branch of arduous mathematics and topology has been created to investigate the quite unexplored theory of the positive maps in Hilbert spaces in view of the assessment of the residual entanglement and of the establishment of more general state-separability criteria [6,7]. In quantum optics a consistent and wide range investigation of these aspects reqires the availability of a universal, flexible source by which two-qubit photon states of any structure could be easily enginerered in a reliable and reproducible way. In this letter we demonstrate that a novel high brilliance spontaneous parametric down conversion (SPDC) source of polarization entangled photons, recently developed in our laboratory, possesses these properties [8–10]. Tunable Werner states [11] and maximally entangled mixed states (MEMS) [12,13] have been synthesized by this source. Their properties have been fully characterized by quantum tomography [14] and Bell's inequalities test [15].

A detailed description of our high brilliance source of entanglement has been recently published [8,10]. It consists of a high stability single arm interferometer where the polarization entangled state $|\Phi\rangle = 2^{-1/2}(|HH\rangle + e^{i\phi}|VV\rangle)$, expressed in the horizontal (H) and vertical (V) basis, arises from the superposition of the SPDC emission cones ($\lambda = 727.6$ nm), generated by a thin type I β-BaB_2O_4 (BBO) crystal which is excited in two opposite directions (left and right) by a backreflected cw Ar^+ laser beam ($\lambda_p = 363.8$ nm) (cf. Fig. 1). The crucial element for generation of entanglement is represented by the "d section," where both the pump and the photon pairs are reflected by a spherical mirror M with radius $R = 15$ cm, highly reflecting both λ and λ_p, placed at a distance $d = R$ from the crystal. In this section a zero-order $\lambda/4$ wave plate (wp) performs the $|HH\rangle \rightarrow |VV\rangle$ transformation on the two-photon states belonging to the left cone while leaving in its original polarization state the pump beam ($\lambda_p = \lambda/2$). The phase ϕ is reliably controlled by micrometric displacements of M. A positive lens transforms the overall conical emission distribution into a cylindrical one whose transverse circular section, spatially selected by an annular mask with diameter $D = 1.5$ cm and width $\delta = 0.07$ cm, identifies the entanglement ring (e ring). After division of the ring along a

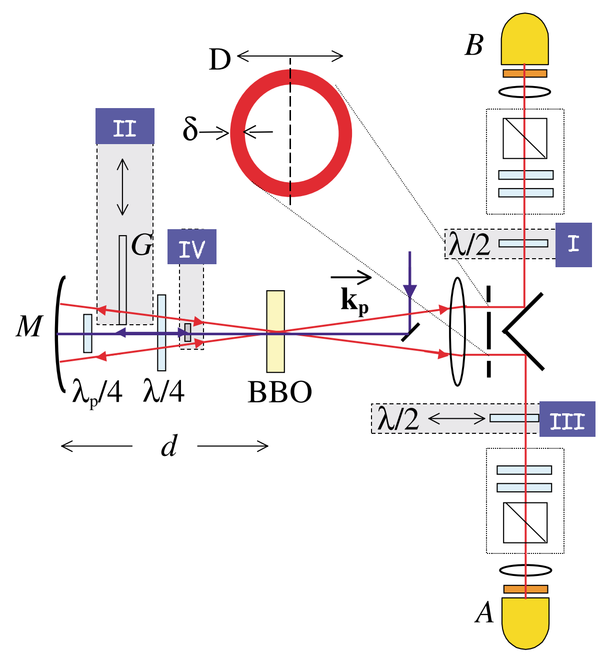

FIG. 1 (color online). Layout of the high brilliance source of polarization entanglement. The two dashed boxes located before detectors A and B represent the system for tomogaphic reconstruction of the produced states.

vertical axis, the two resulting equal portions are detected at sites A and B within a bandwidth $\Delta\lambda = 6$ nm, which corresponds to the coherence time $\tau_{\rm coh} \approx 140$ fsec. The nonlocal nature of the state $|\Phi^-\rangle$ ($\phi = \pi$) has been recently demonstrated by a Bell inequality experiment which gave a violation of $213 - \sigma$ with respect to the limit value $S = 2$ implied by local realistic theories [8,15]. Furthermore, the optional insertion of a zero-order $\lambda p/4 w p i n t h e$d section of the source, intercepting only the UV beam (cf. Fig. 1) allows the engeneering of tunable nonmaximally entangled states, a class of states by which the nonlocality of quantum mechanics can be tested without inequalities [8,16].

By this source, all possible bipartite states in 2×2 dimensions with tunable degree of mixedness and a minimum expense of photons can be created. This expresses its character of flexibility and universality, as is demonstrated in the case of the creation of Werner states and MEMS.

Werner states and MEMS are two particular classes of mixed states whose density matrix, expressed in the $|HH\rangle$, $|HV\rangle$, $|VH\rangle$, $|VV\rangle$ basis can be written as

$$\rho = \begin{pmatrix} A & 0 & 0 & 0 \\ 0 & B & C & 0 \\ 0 & C & B & 0 \\ 0 & 0 & 0 & D \end{pmatrix}, \qquad (1)$$

Werner states [11] $\rho_W = p|\Psi^-\rangle\langle\Psi^-| + \frac{1-p}{4}\mathrm{I}_4$ consist of a mixture of a pure singlet state $|\Psi^-\rangle = 2^{-1/2}\{|HV\rangle - |VH\rangle\}$ with probability p ($0 \leq p \leq 1$) and a fully mixed state with probability $1 - p$, expressed by the unit operator I_4 defined in the 4-dimensional Hilbert space. The density matrix ρ_W is given by the parameters $A = D = \frac{1}{4}(1 - p)$, $B = \frac{1}{4}(1 + p)$, $C = -p/2$. Depending on the singlet weight p, Werner states may be entangled ($p > \frac{1}{3}$) or separable ($p \leq \frac{1}{3}$) [6,11]. Moreover, they possess a highly conceptual and historical value because, in the probability range $[1/3 < p < 1/\sqrt{2}]$, they do not violate any Bell's inequality in spite of being in this range nonseparable entangled states, precisely, negative partial transpose (NPT) states [6,17]. These states are also important for QI, since they model a decoherence process occurring on a singlet state traveling along a noisy channel [18].

MEMS are to be considered as peculiar ingredients of modern quantum information because their entanglement cannot be increased by any unitary transformation [12,13]. Since the Werner states share this property they can be assumed to belong to the broader class of generalized MEMS [19]. The MEMS generated and tested by our method are the ones that own the maximum value of *tangle* T allowed for a given value of *linear entropy* S_L [19,20]. The density matrix $\rho_{\rm MEMS}$ is represented by Eq. (1) with the parameters $A \equiv [1 - 2g(p)]$, $B \equiv g(p)$, $C \equiv -p/2$, $D \equiv 0$. We define $g(p) = p/2$ for $p \geq \frac{2}{3}$ and $g(p) = \frac{1}{3}$ for $p < \frac{2}{3}$.

A simple *patchwork* technique, consisting of a sequence of different local operations performed by four devices labeled with numbers I, II, III, IV in Fig. 1, is at the basis of the creation of Werner states and MEMS. They are (I) A $\lambda/2$ wp inserted in front of detector B performs the $|\Phi^-\rangle \rightarrow |\Psi^-\rangle$ transformation. (II) A 200 μm thick glass-plate G inserted in the d section introduces a decohering fixed time delay $\Delta t \gg \tau_{\rm coh}$ and spoils the indistinguishability between the intercepted portion of the radiation cones. The corresponding contribution to the nondiagonal elements of ρ are set to zero, while the nonintercepted sector expresses the entangled state contribution to ρ. (III) A $\lambda/2$ wp, inserted in the semicylindrical photon distribution reflected by the beam-splitting prism towards the detector A, rotates by $\pi/2$ the polarization of the photons belonging to the intercepted portion of the ring. (IV) A triangular opaque screen erases a portion of the left cone while the corresponding sector of the right cone is kept unaltered.

Any Werner state can be realized over its full range of values, from $p = 0$ ($\rho_W = \frac{1}{4}\mathrm{I}_4$) to $p = 1$ ($\rho_W = |\Psi^-\rangle\langle\Psi^-|$) by following the sequence: (I), (II), (III) [9,10]. The singlet contribution to ρ is given in this case by the nonintercepted sector A, while the sectors B + C, with B = C, determine the mixed-state contribution [Fig. 2(a)].

Figures 3(a)–3(c) show the density matrix plots of three Werner states reconstructed by quantum tomography. The experimental values of several Werner states are plotted in the (S_L, T) plane [12,13] (Fig. 4) with good agreement between experimental data and the theoretical plot. We have recently tested the nonseparability condition for Werner states by a powerful entanglement witness method for polarized photons based on few local measurements [10].

A further example concerns the generation of MEMS [9]. The creation of MEMS depends on the singlet weight, either $p < \frac{2}{3}$ or $p \geq \frac{2}{3}$. The $p < \frac{2}{3}$ case is realized by the sequence (III), (IV), (II). In this case D = E + F for the

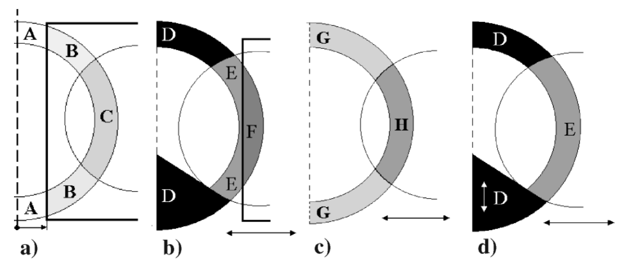

FIG. 2. (a) Partition of the (half) e ring for Werner states production. (b)–(d) Partition of the (half) e ring for MEMS production: (b) left cone ($0 \leq p \leq \frac{2}{3}$), (c) right cone, (d) left cone for $\frac{2}{3} \leq p \leq 1$.

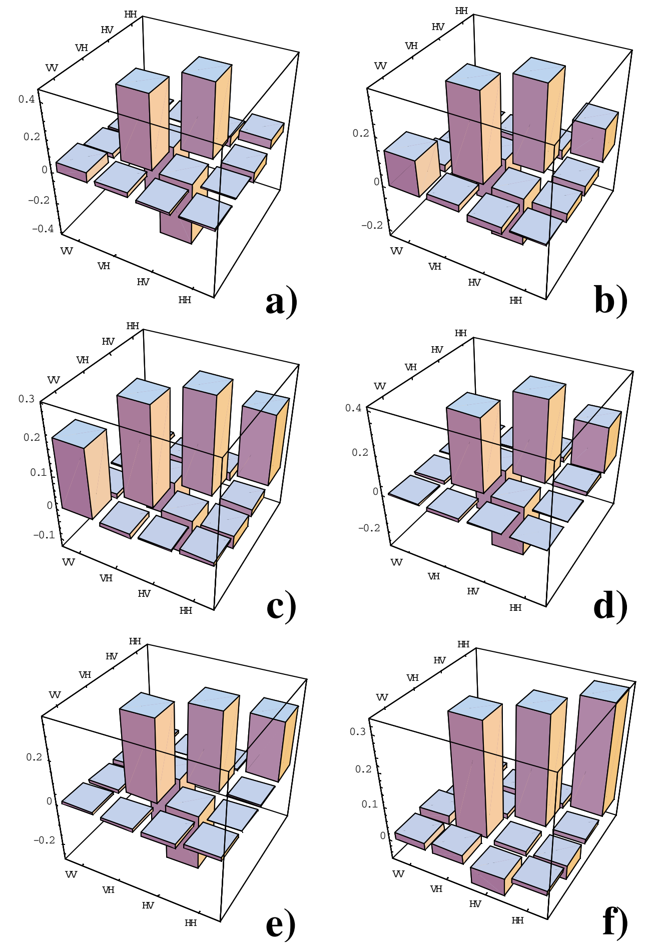

FIG. 3 (color online). Tomographic reconstruction (real parts) of three Werner states (a)–(c) and three MEMS (d)–(f) with singlet weight, $p = 0.82$ (a), $p = 0.47$ (b), $p = 0.27$ (c), $p = 0.77$ (d), $p = 0.45$ (e), and $p = 0$ (f). The imaginary components are negligible.

left cone and G = H for the right cone [Figs. 2(b) and 2(c)]. Sector D, corresponding to half of the left cone ring, is erased, while sector G of the right cone is kept unaltered. Decoherence is induced on a portion F of sector E (left cone) with weight $1 - p$. By transverse displacement of G, p is continuously tuned in the range $0 \leq p < \frac{2}{3}$. The larger the F contribution, the larger the decoherence of the MEMS. In the case $p \geq \frac{2}{3}$ only the devices (III) and (IV) are necessary. The singlet contribution for the right and left cones is given by the sectors H and E and can be tuned in the range $\frac{2}{3} \leq p < 1$ by fine adjustment of the $\lambda/2$ wp [Figs. 2(c) and 2(d)]. For each value of p the vertical position of the opaque screen is adjusted in order to erase the contribution of sector D.

An accurate experimental production of MEMS is particularly severe because of the critical requirements needed for operating on the very boundary between the allowed and the forbidden region of the (S_L, T) plane [12,13]. In this sense any lack of correlation within the singlet contribution limits the quality of MEMS. One of the main limitations to the coherence of the state is given by spatial walk-off occurring in the BBO and in the $\lambda/4$ wp. This effect is reduced by lowering the aperture angle of the emission cone. For this experiment the mask diameter was $D = 0.75$ cm. The results shown in Figs. 3(d)–3(f) reproduce graphically the real components of three different ρ_{MEMS} structures. Several MEMS generated by this technique are plotted in the (S_L, T) plane and compared with the theoretical line (Fig. 4). An alternative realization of MEMS implementing a partial-polarizer based filtering concentration technique, which occurs with a larger expense of photons, has been recently given [20].

We have investigated the nonlocal properties of generalized MEMS by following a unifying variational approach, based on the quantitative test founded by John Bell to verify the completeness of quantum mechanics. In the case of two photons with polarization $\vec{\pi}_1$ and $\vec{\pi}_2$, local realism implies the following inequality: $-2 \leq S = P(\hat{u}_1; \hat{u}_2) - P(\hat{u}_1; \hat{u}_2') + P(\hat{u}_1'; \hat{u}_2) + P(\hat{u}_1'; \hat{u}_2') \leq 2$, where $\hat{u}_1$ and $\hat{u}_2$ are unitary norm vectors related to the angular coordinates (Θ_1, Φ_1) and (Θ_2, Φ_2) in the Bloch sphere and $P(\hat{u}_1; \hat{u}_2)$ is the correlation function, $P(\hat{u}_1; \hat{u}_2) = \langle(\hat{u}_1 \cdot \vec{\pi}_1)(\hat{u}_2 \cdot \vec{\pi}_2)\rangle$.

For any state with density matrix expressed by Eq. (1), with $C = -p/2$, the following property holds: $S = (1 - 4B)F_1 - pF_2$, where F_i $(i = 1, 2)$ are proper functions of the $\hat{u}_i$ vectors. An extremal point for S is found by setting $\frac{\partial S}{\partial \Theta_i} = 0$ and $\frac{\partial S}{\partial \Phi_i} = 0$. It is found that F_1 vanishes when the first condition is fullfilled, i.e., by setting $\Theta_1 = \Theta_2 = \Theta_1' = \Theta_2' = \frac{\pi}{2}$, and S can be rewritten as

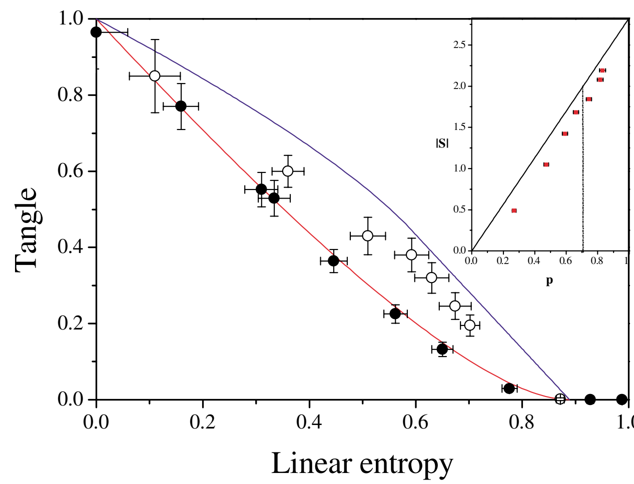

FIG. 4 (color online). Experimental Werner states (solid circles) and MEMS (open circles) plotted in the (S_L, T) plane. Upper line represents the theoretical curve for MEMS, lower line is the expected curve for Werner states. Inset: plot of the Bell parameter $|S|$ as a function of weight p for Werner states. The expected straight line $|S| = 2\sqrt{2}p$ is also reported. The vertical dashed line represents the boundary for CHSH inequality violation.

$$S = -p[\cos(\Phi_1 - \Phi_2) - \cos(\Phi_1 - \Phi_2') + \cos(\Phi_1' - \Phi_2) + \cos(\Phi_1' - \Phi_2')]. \quad (2)$$

This parameter attains its minimum value $-2\sqrt{2}p$ for $\Phi_1 = 0$, $\Phi_2 = \frac{\pi}{4}$, $\Phi_1' = \frac{\pi}{2}$, $\Phi_2' = 3\frac{\pi}{4}$. The corresponding vectors lie on the circular polarization plane of the Bloch sphere. As a consequence it is obtained the very general result that Bell inequality violation is observed for states with $p > 1/\sqrt{2}$, regardless the diagonal terms, i.e independently of the state symmetry [21]. Because of the rotational invariance properties of the Werner states [11], we can rotate the plane of circular polarizations into the one of linear polarizations, by simply exchanging $\Phi \to \Theta$ and $\Phi' \to \Theta'$ in (2). Therefore, we obtained the well-known result that, in the range $p \in [1/\sqrt{2}, 1/3]$, the Werner states, although not separable, do not violate the Clauser-Horne-Shimony-Holt (CHSH) inequality [6,17]. Hence we can adopt the standard measurement setting of the Bell–CHSH inequalities [15].

The three Werner states shown in Figs. 3(a)–3(c) correspond to the following conditions: state (a) is not separable and violates the CHSH inequality ($|S| = 2.0790 \pm 0.0042$); state (b) is not separable and does not violate the CHSH inequality ($|S| = 1.049 \pm 0.011$); state (c) is separable ($|S| = 0.4895 \pm 0.0047$). The results corresponding to a systematic test of the Bell–CHSH inequalities for several Werner states are shown in the inset of Fig. 4. The experimental data, placed under the theoretical straight line, indicate a still nonperfect correlation within the singlet region A in Fig. 2(a).

In this Letter we have presented a reliable and efficient technique for the production of Werner states and MEMS, two families of mixed states of wide interest in quantum information, in order to study decoherence processes. We used polarized photons generated by a high brilliance source of entanglement which is completely tunable to achieve any bipartite two-photon state. These states have been characterized by quantum tomography. The nonlocal properties of these states have been investigated both experimentally and theoretically.

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